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A TURBULENCE MODEL FOR NON-NEWTONIAN POWER-LAW FLUIDS IN DUCTS

Daniel Onofre de Almeida Cruz
Mechanical Engineering Department-CT/Universidade Federal do Pará-UFPA
João Nazareno Nonato Quaresma
Chemical Engineering Department-CT/Universidade Federal do Pará-UFPA
Carlos Edilson Maneschy
Mechanical Engineering Department-CT/Universidade Federal do Pará-UFPA
Emanuel Negrão Macêdo
Mechanical Engineering Department-CT/Universidade Federal do Pará-UFPA
Campus Universitário do Guamá, Rua Augusto Corrêa, 01
66075-900 - Belém, PA, Brasil

Abstract. *The problem of the turbulence modeling for non-newtonian power-law fluids inside ducts appears in many fields of chemical, food and mechanical engineering. In spite its importance, very few articles can be found in the literature that analyze the characteristics of the turbulence phenomenon for non-newtonian power-law fluids. In this context, the purpose of this work is to analyze the problem of the turbulence modeling of such fluids. First, the Kolmogorov characteristic parameters for the case of power-law fluids is obtained. The characteristic dissipation length, is showed to be a function of the power-law index (n) and reduces to the newtonian case when n = 1. In parallel, a turbulence algebraic model is proposed, in which an extension of the Van-Driest damping function for the power-law fluids is developed. A numerical solution for the hydrodynamically developed turbulent equation of motion flow in ducts is obtained. A comparison with experimental data and an empirical correlation for turbulent flow of non-newtonian power-law fluids is made, showing good agreement.*

Keywords: *Turbulence model, Power-law fluids, Pipe flow.*

1. INTRODUCTION

The fluid dynamics of turbulent non-newtonian flow in ducts is of special interest in many engineering applications. The non-newtonian behavior of certain materials or fluids is encountered in several industrial applications, namely, polymer extrusion, greases, toothpaste and drilling muds. A discussion of several uses of non-newtonian fluids is pointed in Macêdo and Quaresma (1997), Quaresma and Macêdo (1998), Macêdo et al. (1998a) and Macêdo et al. (1998b). In spite of its importance, very few works can be found in the literature that analyze in detail the behavior and the characteristic parameters of such turbulent flows, like the Kolmogorov dissipation length.

In a recent work, Kostic (1994) pointed out that turbulent flow of non-newtonian fluids is not quite well understood, because the classical isotropic fluid mechanics is not applicable to very complex fluids, and because the turbulence phenomenon is also not well understood, even for newtonian fluids; so that many questions about these fluids remain unanswered.

The description of non-newtonian fully-developed turbulent flows in ducts for engineering ends, is basically restricted to some empirical correlations (Dodge and Metzner, 1959). Recently, Malin (1997) has employed the k-ε turbulence model to describe the fully-developed power-law fluid flow in ducts, using the exponential Van Driest damping function to characterize the vanishing of the turbulent shear stress in the viscous sub-layer. This approach assumes that the viscous action of power-law fluids in the very near wall region, is similar to the viscous action of newtonian fluids, but no analytical proof of that statement was furnished by the author.

In this work, a version of the turbulent Kolmogorov's dissipation length for non-newtonian power-law fluids will be obtained, using the same dimensional arguments applied in the original Kolmogorov's work for the newtonian fluids. This generalized dissipation length is a function of the power-law index (n) and reduces to the newtonian case when this parameter is equal to the unity. A new mixing length turbulence model for power-law fluids will also be deduced. In this model the near wall Van Driest (1965) damping function will be modified to take into account the influence of the power-law index on the disappearance of the turbulent shear stress in the viscous sub-layer near the wall. The model reduces to the Cebeci-Smith (1969) one when the power-law index approaches the unity. The resulting equations will be solved numerically for the case of pipe flow. The resulting Fanning friction factor will be compared with experimental data and with a semi-empirical correlation, showing that the proposed formulation can describe accurately the phenomenon.

2. THE KOLMOGOROV NON-NEWTONIAN DISSIPATION LENGTH

The energy dissipation by viscous effects occurs at the smallest eddies whose Reynolds number is comparable to the unity. For non-newtonian fluids this fact can be described by the following equation:

$$\frac{K}{\rho}\frac{u_\ell^{n-2}}{\ell^{n}} = 1 \tag{1}$$

In equation above ρ is the fluid density, ℓ is the order of magnitude of the size of the smallest eddy, u_ℓ is the order of magnitude of its velocity and K represents the consistency index. The power-law index is represented by n.

Since the energy is dissipated into heat, in order to maintain the steady-state situation, it is necessary an external source of energy to supply the larger eddies. The energy passes from the larger eddies to the smaller ones with practically no dissipation occurring in this process. Therefore, the energy flux obtained from the external source, which is ultimately the energy dissipated in the smaller eddies, can be determined only by those quantities that characterize the larger eddies. These are the dimension L and the velocity U. There is only one combination of these quantities that furnishes the required dimension of the energy dissipation, thus we find (Landau and Lifshitz, 1987):

$$\varepsilon \approx \frac{U^3}{L} \tag{2}$$

where ε represents the energy dissipation per unit of time per unit of mass of fluid.

The energy dissipation can also be described using the characteristic quantities of the smaller eddies which are ℓ, u_ℓ, K and ρ. From these four quantities we can only form one having the dimension of energy flux namely,

$$\varepsilon \approx \frac{K}{\rho}\frac{u_\ell^{n+1}}{\ell^{n+1}} \tag{3}$$

Combining the equations (1), (2) and (3) we can obtain an expression for the Kolmogorov's dissipation length for power-law fluids:

$$\frac{\ell}{L} = Re_{ap}^{-3/(2(n+1))} \tag{4}$$

where

$$Re_{ap} = \frac{L^n U^{2-n}}{K/\rho} \tag{5}$$

Equation (4) represents the relation between the size of the smallest eddies, and the large eddies Reynolds number. This relation reduces to the classical Kolmogorov's expression when the power-law index approaches the unity.

3. THE NON-NEWTONIAN TURBULENCE MODEL

The result that will be deduced here, is a generalization of the Cebeci–Smith (1969) mixing length model. To obtain the turbulence model, we will follow here the same approach used by Van Driest (1965) to develop a damping function that describes how the turbulent stresses behave in the viscous sub-layer of power-law fluids. In his work Van Driest tried to represent the physics of the damping of the turbulent eddies by likening that flow to the laminar flow near an oscillating wall in a fluid otherwise at rest. That problem has been solved by Stokes (1851) for newtonian fluids. For power-law fluids the phenomenon is described by the following equation:

$$\frac{\partial u}{\partial t} = \frac{1}{Re_{ap}}\frac{\partial}{\partial y}\left[\mu(y)\frac{\partial u}{\partial y}\right], \quad y > 0 \ ; \quad t > 0 \tag{6.a}$$

$$u(t, y = 0) = U_o \cos(\omega t) \quad ; \quad u(t, y \to \infty) = 1 \tag{6.b,c}$$

where

$$\mu(y) = \left|\frac{\partial u}{\partial y}\right|^{n-1} \qquad (6.d)$$

In order to obtain the turbulence model we assume the separation of function u(t, y) into two functions in the form

$$u(t, y) = f(y)\, g(\alpha_o t + \alpha_1 \ell n(f)) = f(y)\, g(\phi) \qquad (7.a)$$

where α_o and α_1 are constants.

The substituting of equation (7.a) into equation (6.a) yields

$$\alpha_o \, Re_{ap}\, f(y) g'(\phi) = \mu'(y) f'(y)\left[g(\phi) + \alpha_1 g'(\phi)\right] + \\ + \mu(y)\left(f''(y)\left[g(\phi) + \alpha_1 g'(\phi)\right] + \alpha_1 \frac{f'^2(y)}{f(y)}\left[g'(\phi) + \alpha_1 g''(\phi)\right]\right) \qquad (7.b)$$

where

$$\mu(y) = \left|\; f'(y)\left[g(\phi) + \alpha_1 g'(\phi)\right] \;\right|^{n-1} \qquad (7.c)$$

We will now seek the solution of equations (7) assuming that f can be described by the following expression:

$$f = (a + by)^p \qquad (8.a)$$

where a, b and p are constants

Substituting equation (8.a) into (7.a) and applying the boundary conditions we have:

$$p = -\frac{1+n}{1-n} \quad ; \quad a = 1 \qquad (8.b, c)$$

The constant b is obtained assuming that the equation (8.a) must reproduce the exponential behavior of the Van Driest model as n approaches the unity, thus we find:

$$b = \frac{1}{26}\left|\frac{1-n}{1+n}\right| \qquad (9)$$

The equation (8.a) can now be rewritten as:

$$f = \left(1 + \frac{1-n}{1+n}\frac{y}{26}\right)^{-\frac{1+n}{1-n}} \qquad (10)$$

Equation (8.a) can now be used to compose the turbulence model, by substituting the Van Driest damping function in the classical mixing length formulation, is furnished the following equation:

$$\tau_{uv} = \left[\kappa n y \left(1 - \left(1 + \left| \frac{1-n}{1+n} \right| \frac{y^+}{26} \right)^{-\left|\frac{1+n}{1-n}\right|} \right) \right]^2 \left| \frac{\partial u}{\partial y} \right| \frac{\partial u}{\partial y} \tag{11}$$

In equation above τ_{uv} represents the turbulent shear stress, κ is the Von Kàrman constant (=0.41) and $y^+ = y u_\tau / \nu$ where u_τ is the friction velocity.

4. NUMERICAL ANALYSIS

The mathematical formulation for fully-developed turbulent power-law fluids in circular ducts in dimensionless form is written as:

$$\frac{1}{R}\frac{d}{dR}\left(RF(R)\frac{dU}{dR} \right) = -Re_{ap}\, f_f / 2^n, \quad 0 < R < 1 \tag{12.a}$$

$$\left. \frac{dU}{dR} \right|_{R=0} = 0 \quad ; \quad U(1) = 0 \tag{12.b, c}$$

where the following dimensionless groups were employed in equations (12) above

$$R = \frac{r}{r_w}; \quad Re_{ap} = \frac{\rho u_m^{2-n} D_h^n}{k}; \quad U(R) = \frac{u(r)}{u_m}; \quad f_f = \left(-\frac{dp}{dz} \right) \frac{D_h}{2\rho u_m^2} \tag{13.a-d}$$

with $D_h = 2r_w$ being the hydraulic diameter and Re_{ap} and f_f the apparent Reynolds number and Fanning friction factor, respectively.

The function F(R) for the case of hydrodynamically developed turbulent flow of power-law fluids is given by:

$$F(R) = \left| \frac{dU}{dR} \right|^{n-1} + \frac{Re_{ap}}{2^n} \ell(R) \left| \frac{dU}{dR} \right|; \quad \ell(R) = \left[\kappa n (1-R) \left(1 - \left(1 + \left| \frac{1-n}{1+n} \right| \frac{y^+}{26} \right)^{-\left|\frac{1+n}{1-n}\right|} \right) \right]^2 \tag{14.a, b}$$

where y^+ is the wall dimensionless coordinate described below:

$$y^+ = \frac{1}{2}\left[\left(2\sqrt{f_f / 8} \right)^{2-n} Re_{ap} \right]^{1/n} (1-R) \tag{14.c}$$

Equation (12) can be rewritten in the resulting form:

$$\frac{1}{R}\frac{d}{dR}\left[RF(g)g(R)\right] = -Re_{ap}\, f_f \,/\, 2^n \; ; \quad g(R) = \frac{dU}{dR} \tag{15}$$

which is readily integrated to furnish the following nonlinear equation for g(R)

$$F(g)g(R) = -\frac{Re_{ap}\, f_f}{2^{n+1}} R \tag{16.a}$$

Equations for U(R) and the equation that satisfies the average flow velocity, in terms of the g(R), are given in the following form

$$U(R) = U_{max} + \int_0^R g(R)dR \quad ; \qquad \int_0^R R^2 g(R)dR = -1 \tag{16.b, c}$$

Expressions (16.a-c) represent a system of equations for g and f_f. Initially the system is analyzed dividing the domain (0<R<1) in N parts. Then equation (16.a) is solved numerically as an ordinary transcendental equation using the routine ZREAL from IMSL Library (1989) for each value of the independent variable (R) and for a initial guess of f_f near zero ($f_f = 10^{-6}$). The result is substituted in the right side of equation (16.c); if the relation (16.c) is not satisfied, with a tolerance of 10^{-8}, the procedure is repeated for another guess of the friction factor f_f, obtained by the addition of a small increment in the preceding value. If relation (16.c) is satisfied, the velocity U(R) is calculated numerically through equation (16.b).

5. RESULTS AND DISCUSSION

In Table 1 the Fanning friction factor obtained from equation (12) is compared with the experimental results from Yoo (1974) and with the empirical correlation of Dodge and Metzner (1959). The calculated results are in good agreement with both situations, showing that the present formulation can be used in engineering calculations providing high performance results.

Some discrepancy between the friction factor calculated by the present theory and the experimental data can be found for lower values of the apparent Reynolds number. This can be explained considering that the present work assumes a fully-developed flow, which may have not been attained in the initial sections of the experiment.

In Table 2 a comparison of the Fanning friction factor predicted by the present theory, with the Dodge and Metzner (1959) correlation, for various apparent Reynolds number and various power-law indices is made. As can be seen, the calculated friction factor assumes its maximum value for n = 1.5, also a maximum difference between the two compared values is observed. This difference occurs due to the fact that the Dodge and Metzner empirical correlation was developed for pseudoplastic fluids (n<1), yielding inadequate results for dilatant fluids (n>1).

Table 1 – Fanning friction factor computed from the present model and comparison with results from the literature.

n	Re_{ap}	f (+)	f (*)	f (#)
0.8924	10326.402	7.2599E-3	6.6431E-3	6.6010E-3
	21246.379	5.7943E-3	5.5317E-3	5.6580E-3
	31445.527	5.1872E-3	5.0411E-3	5.1470E-3
	39279.405	4.8860E-3	4.7912E-3	4.9200E-3
	46926.098	4.6641E-3	4.6045E-3	4.7150E-3
0.8382	3473.1636	9.8550E-3	8.4098E-3	8.0080E-3
	8130.7993	6.9903E-3	6.5603E-3	6.4950E-3
	13498.821	5.8726E-3	5.7343E-3	5.7450E-3
	16811.479	5.4750E-3	5.4244E-3	5.4690E-3
	20141.228	5.1786E-3	5.1877E-3	5.2320E-3
0.9147	44315.107	4.9919E-3	4.8070E-3	4.4000E-3
0.8724	32124.345	4.9164E-3	4.8782E-3	4.6200E-3
0.8412	25743.249	4.8503E-3	4.9119E-3	4.6740E-3
1.000	18060.000	7.6631E-3	6.6165E-3	6.8600E-3
1.000	75195.000	5.3331E-3	4.7813E-3	4.8780E-3
1.000	151371.00	4.5791E-3	4.1487E-3	4.1103E-3

(+) Present work, (*) Dodge and Metzner (1959), (#) Experimental: Yoo (1974)

Table 2 – Fanning friction factor computed from the present model for various n and Re_{ap}.

$f \times 10^{+3}$						
Re_{ap}	n = 0.5		n = 1.0		n = 1.5	
10000.	2.4737^{+}	3.2336^{*}	9.1812^{+}	7.7011^{*}	19.863^{+}	12.582^{*}
50000.	1.3078	2.1823	5.8683	5.2194	12.847	8.4961
100000.	1.1330	1.8819	5.0034	4.5072	11.164	7.3279

(+) Present work, (*) Dodge and Metzner (1959)

Figures 1 to 3 compare the predicted mean velocity profile with those obtained by using the universal velocity profile for a power-law fluid in turbulent circular pipe flow suggested by Clapp (1961), for different apparent Reynolds number and various power-law indices. In general there are good agreements among the calculated profiles and those obtained through the Clapp profiles, although the latter does not predict the zero velocity gradient profile at the centerline.

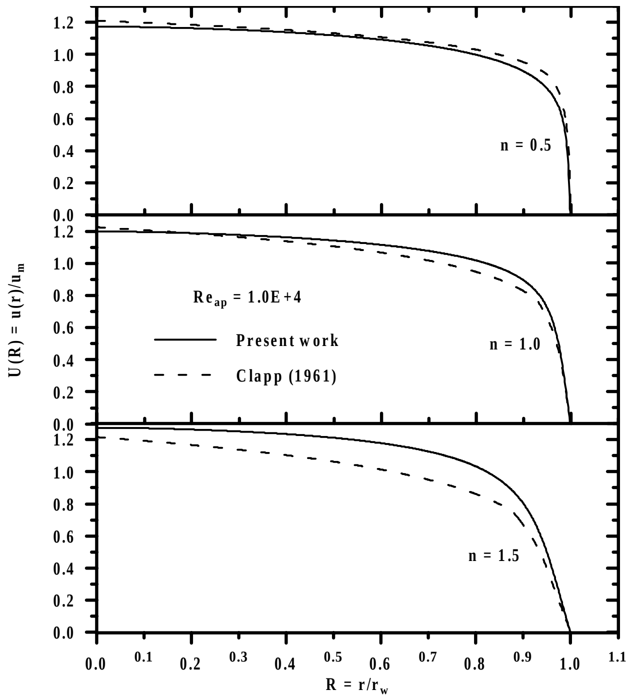

Figure 1. Effect of the parameter n on the turbulent velocity profiles for $Re_{ap} = 10^4$.

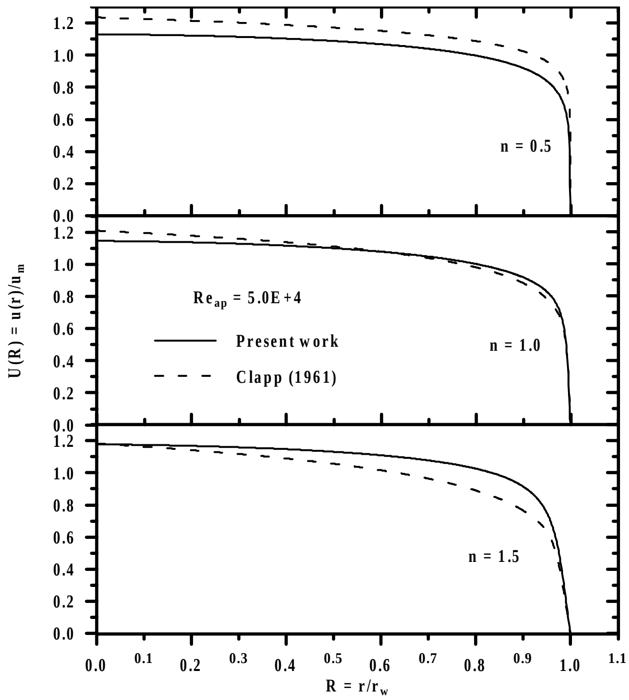

Figure 2. Effect of the parameter n on the turbulent velocity profiles for $Re_{ap} = 5x10^4$.

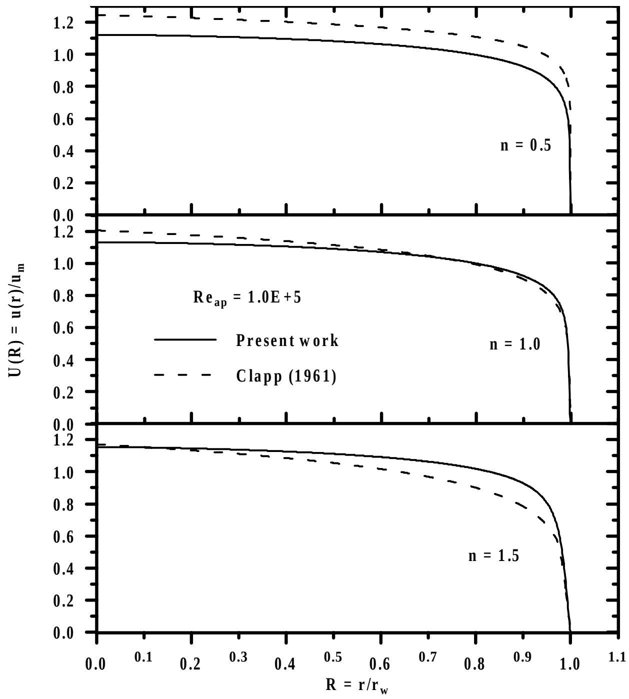

Figure 3. Effect of the parameter n on the turbulent velocity profiles for $Re_{ap} = 10^5$.

6. CONCLUSIONS

The problem of the turbulence modeling for non-newtonian power-law fluids has been analyzed. The Kolmogorov characteristic dissipation length for the case of power-law fluids was proposed and it was shown that this characteristic dissipation length is a function of the power-law index (n) and reduces to the newtonian case when n = 1. In addition, a new algebraic turbulence model was also deduced, and an extension of the Van Driest damping function for the power-law fluids was presented. A series of numerical computation has been performed to calculate hydrodynamically developed turbulent flow. The velocity profile and the Fanning friction factor for various apparent Reynolds numbers, with different values for the power-law index (n) were calculated. The model proposed here shows good agreements with experimental data and empirical correlation for turbulent flow of non-newtonian power-law fluids.

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